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Discussion of
PAVLOVSKY'S THEORY FOR PHREATIC LINE AND SLOPE STABILITY

by K.P. Karpoff, A.M., ASCE
(Proc. Sep. 386)

HAMILTON GRAY,[1] M. ASCE.—This discussion is limited to comments on the second portion of Mr. Karpoff's paper which appears under the heading, "Stability of Slopes". Certain modifications appear to be appropriate in the interest of accuracy of analysis, although the actual numerical discrepancies which are involved do not appear to be serious. The deterioration of the face of a slope by seepage water in the manner contemplated by Mr. Karpoff occurs not only in artificial earth dams and embankments, but also in natural slopes where the face of the slope intercepts the natural position of the ground water surface. Usually this process of deterioration is observed after a slope has been newly formed or altered in connection with construction, since the deterioration, if allowed to continue unchecked, will in time create a stable situation following successive erosion and slumping of slope material.

Figure #1 shows a flow net illustrating the character of seepage flow through homogeneous and isotropic material in the immediate vicinity of the downstream toe of an earth dam, or near the foot of a natural earth slope, when immediately underlain by impervious material. The hydraulic gradient is equal to the head loss, Δh, represented by any two consecutive equipotential curves divided by the distance, $\Delta \underline{1}$, measured along a flow line, between these curves. The direction of seepage flow is normal to the equipotential curves in isotropic materials. The seepage therefore is in the direction of the slope surface at point "M" where the phreatic line intersects the downstream slope while at point "N" the direction of seepage is horizontal. The magnitude of the hydraulic gradient at point "M" is equal to sin α and its magnitude at point "N" is tan α. The seepage force, which represents the effect of fluid friction upon the porous material through which it flows, is numerically equal to the product of hydraulic gradient by unit weight of water, and has the same direction as the flow.

Figure # 2 illustrates the forces exerted upon the mineral constituents forming two rectangular blocks of soil at the downstream face of the dam, one block being located at point "M", of Figure #1 and the other at point "N". The solid matter is acted upon by its buoyant weight, "W", and by the seepage force exerted by the flowing water, "F". For equilibrium it is necessary that the net force which urges the block of soil down the slope be no greater than the available resistance to such motion offered by the soil supporting the block on a plane parallel to the face of the slope. The buoyant weight of solid matter comprising either of these blocks of material is given by the following equation:

1. $$W = \gamma_w (S-1)(1-n) LD = \gamma_w \frac{S-1}{1+e} LD = \gamma_w S_b LD.$$

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wherein e is the void ratio, $S_b = (S-1)(1-n)$ is the buoyant specific gravity of the solid material as given by the quotient of buoyant unit weight by unit weight of water, L and D are the dimensions of the block of earth, and the other symbols are defined in Mr. Karpoff's paper.

This weight, W, is conveniently decomposed into two components, one normal to the slope given by the equation:

2. $$N = W \cos\alpha = \gamma_w S_b LD \cos\alpha, \text{ and}$$

the other parallel to the slope given by the equation:

3. $$T = W \sin\alpha = \gamma_w S_b LD \sin\alpha.$$

The normal component develops friction which may be available in the supporting soil and hence aids in stabilizing the slope. The tangential component tends to move the prism of material down the slope.

The effect of the seepage force "F" will vary with its direction. In the case of point "M" the full magnitude of the seepage force is directed down the slope, whereas in the case of point "N" the seepage force has one component acting down the slope and a second acting in the opposite direction to the normal component of "W". For these two conditions two different relationships determine the maximum angle at which the slope is stable. For the two points, "M" and "N", analysis of the equilibrium of the force vectors of Figure 2 results in the relationships derived through equations 4 to 7 inclusive:

For Point "M"

4. $$F = \gamma_w LD \sin\alpha.$$

5. $$R = N \tan\phi = \gamma_w S_b LD \cos\alpha \tan\phi.$$

For Equilibrium it is necessary that

6. $$F + T \leqq R$$

or

$$\gamma_w LD \sin\alpha + \gamma_w S_b LD \sin\alpha \leqq \gamma_w S_b LD \cos\alpha \tan\phi.$$

$$(1+S_b)\sin\alpha \leqq S_b \cdot \cos\alpha \tan\phi$$

7. $$\tan\alpha \leqq \frac{S_b}{1+S_b} \tan\phi = \frac{S-1}{S+e} \tan\phi.$$

For Point "N"

4. $$F = \gamma_w LD \tan\alpha.$$

5. $$R = (N - F\sin\alpha)\tan\phi = (\gamma_w S_b LD \cos\alpha \gamma_w LD \sin\alpha \tan\alpha)\tan\phi.$$

For Equilibrium it is necessary that

6. $$F\cos\alpha + T \leqq R$$

or

$$\gamma_w LD \sin\alpha = \gamma_w S_b LD \sin\alpha \leqq \gamma_w LD (S_b - \tan^2\alpha)\cos\alpha \tan\phi.$$

$$(1+S_b)\sin\alpha \leqq (S_b - \tan^2\alpha)\tan\phi \cos\alpha.$$

7. $$\frac{\tan\alpha}{S_b - \tan^2\alpha} \leqq \frac{1}{1+S_b}\tan\phi, \text{ or } \tan\alpha \leqq \frac{S_b - \tan^2\alpha}{1+S_b}\tan\phi.$$

Neither of the relationships given in equations 7 agrees with the equation 22 of Mr. Karpoff's paper. This is because it is assumed in his analysis that the seepage force, while directed parallel to the slope, is given by $F = \gamma_w LD \tan\alpha$. This direction is correct but the magnitude is not for point M, and the magnitude is correct but the direction is not for point N. If the values utilized in Mr. Karpoff's numerical example are introduced into the two equations 7, one obtains for point "M" a maximum stable slope angle of $\alpha = 12^\circ\ 22\ 1/2'$ and at point "N" a maximum stable slope angle of $\alpha = 11^\circ\ 51\ 2/3'$, as contrasted with a slope angle of $12^\circ\ 14\ 1/2'$ by Mr. Karpoff's method. These values do not differ by any great amount and it should be pointed out that as smaller angles of friction are considered and smaller stable slope inclinations determined, the differences in the three methods of computation become smaller and smaller. On the other hand, if the angle of friction is increased, then the discrepancies between the results of the three methods of determining α will also increase. Fig. 3 illustrates graphically the relationship between maximum slope angle, α, and angle of internal friction, ϕ, in accordance with equations 7 and Mr. Karpoff's equation 22.

Mr. Karpoff's equation 25 seems to indicate that the addition of a surface layer of highly pervious material will rapidly increase the stability of a given slope. It would appear however that if the depth, D, of material which tends to move down the slope is very great, the addition of a surface blanket of a given thickness, a, would not be as effective in developing stability as it would were the dimension, D, very small. Considering the equilibrium of forces at point "M" when the face of the slope is surcharged by a free-draining blanket with a unit weight of w and a thickness, d, the vertical force acting upon the soil prism in the face of the slope is given by equation:

8. $$W' = \gamma_w S_b LD + w LD.$$

and the normal and tangential components of this force by equations:

9. $$N' = (\gamma_w S_b LD + wLD) \cos\alpha. \quad \text{and}$$

10. $$T' = (\gamma_w S_b LD + wLD) \sin\alpha.$$

The shearing resistance which develops is given by equation:

11. $$R' = (\gamma_w S_b LD + wLD) \cos\alpha \tan\phi.$$

and the maximum stable slope angle by equation:

13. $$\tan\alpha = \frac{\gamma_w S_b D + wd}{\gamma_w D (1+S_b) + wd} . \tan\phi.$$

It is seen that equation 13 involves both the thickness, d, of the blanket and the depth, "D", to which the slope material tends to slide. Figure 4 illustrates, for the numerical data used by Mr. Karpoff, the effect on maximum stable slope angle, of varying the ratio of blanket thickness, d, to depth of moving slope "D". It is clear that when $\frac{d}{D}$ is negligible the blanket is ineffective whereas when $\frac{d}{D}$ becomes very large α approaches ϕ.

It would appear that Mr. Karpoff's equation 25 would indicate the maximum slope which would be stable if the depth, "D", were equal to one foot. The actual distance, "D" to which the surface may be unstable is somewhat indeterminate, and it would appear that only by means of a flow net similar to that

shown in Figure 1 would it be possible, for a given slope, to make a reasonable estimate of depth, D, which must be stabilized by the surface blanket.

In practice the downstream slope of an earthen dam is ordinarily thoroughly protected against the effect of seepage, even if the embankment is constructed of almost homogeneous material. The use of rock toes and horizontal toe drains to divert the seepage line will, if properly designed, keep the seepage line at a substantial distance from the exposed slope. Where the dam cross-section is composed of non-homogeneous materials, the downstream shoulder is ordinarily of more than ample thickness and permeability to entirely eliminate the need to consider the effect of seepage.

On the other hand, the construction of slopes in natural ground frequently brings about a relocation of the ground water table. Ordinarily no effort is made to stabilize such slopes either through reducing the slope angle to a very small value or by installing drainage systems. It is frequently possible to divert the seepage by means of horizontal drains penetrating inward from the toe of the slope. In other cases, particularly where soils are fine-grained, such drains are not wholly effective unless placed very close together. In many instances the expense of installing such drains is excessive.

The practice of placing a granular blanket on the face of a slope has the additional advantage that it reduces the tendency of the slope to erode under the effects of heavy rainfall, and also aids the slope in resisting deterioration caused by frost action. It is believed that the treatment of excavated slopes by means of such blankets deserves serious consideration in many localities where stabilization by other means is not wholly satisfactory.

JOHN A. FOCHT, Jr.[2], A.M. ASCE.—The author has presented a unique procedure to approximate the location of the phreatic line for a homogenous, isotropic embankment However, a homogenous dam section is unusual in present design. Normally, semi-pervious or pervious shells with or without drainage blankets are incorporated in the design of the embankment to make more effective use of the available borrow materials. In addition, a compacted fill is probably not isotropic.

The procedures for analysis of seepage through dams given by Prof. A. Casagrande in 1935[3] enable an engineer to consider both anisotropic and composite sections. The computation of seepage loss has its greatest potential error involved in the determination and selection of the coefficient of permeability. Therefore, the simplest procedure to estimate the average hydraulic gradient through the average area will be satisfactory. A simple flow net can be sketched in a very few minutes which will be as accurate as the prediction of the embankment permeability. It then appears that the procedure presented is not only limited in application but also is more involved than is necessary for computation of seepage loss.

The method for determining the stability of a slope where seepage is emerging is correct for cohesionless isotropic material. The factor of safety as determined is against the movement of a single particle, but the numerical example given is for a clay. No consideration has been given to the cohesive strength of the clay. Many dams, levees and dikes built of clay material have downstream slopes steeper than 1 vertical to 5 horizontal and have proven to be stable.

2. Associate Eng., Greer & McClelland, Cons. Foundation Engs., Houston, Tex.
3. "Seepage through Dams". Arthur Casagrande, Jrnl. of New England Water Works Association, Vol. LI, n2, June 1937.

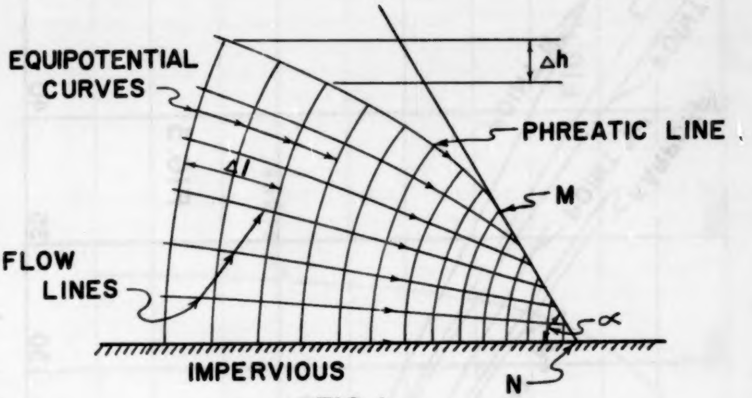

EQUIPOTENTIAL CURVES
Δh
PHREATIC LINE
Δl
M
FLOW LINES
α
IMPERVIOUS
N

FIG. 1

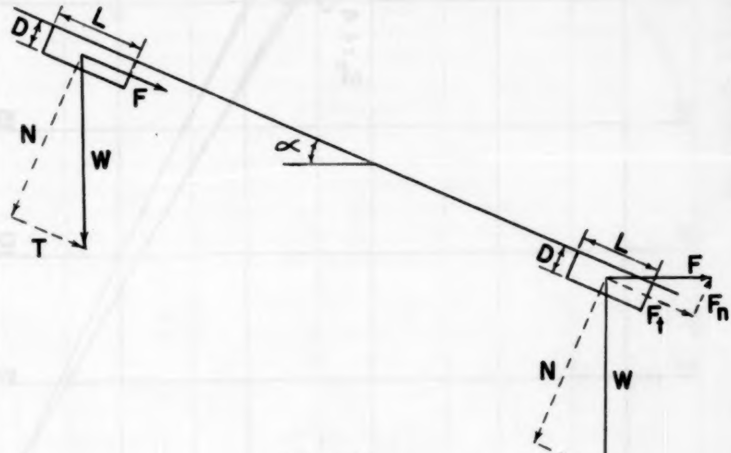

D
L
F
N
W
T
α
D
L
F
F_t
F_n
N
W
T

FIG. 2

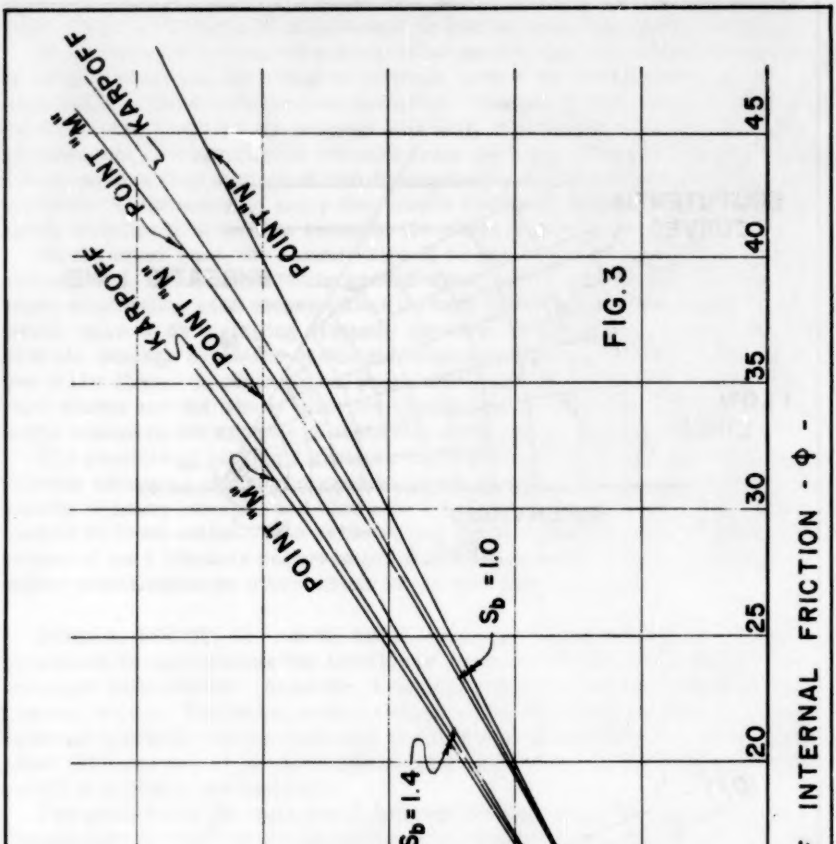
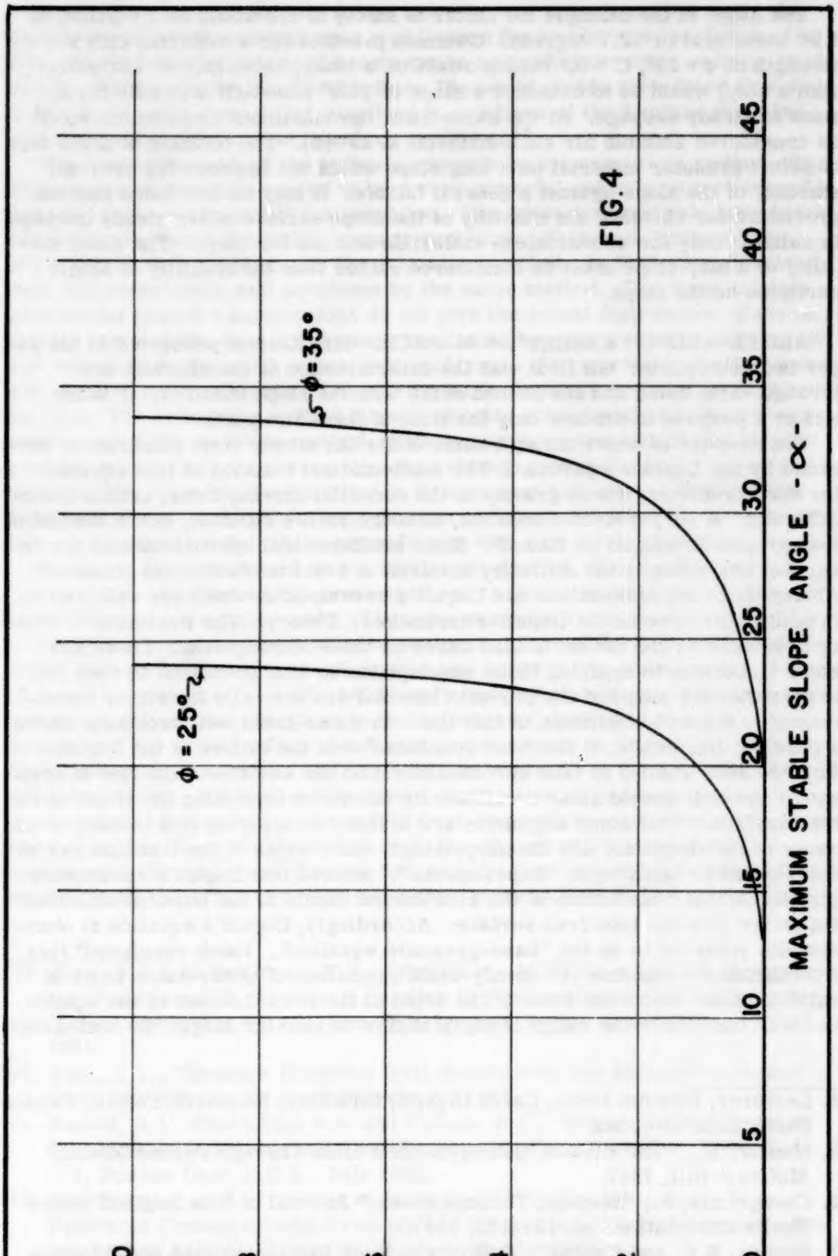

MAXIMUM STABLE SLOPE ANGLE - α -
ANGLE OF INTERNAL FRICTION - φ -
0
5
10
15
20
25
30
35
40
45
POINT "N"
KARPOFF
POINT "M"
POINT "N"
KARPOFF
POINT "M"
S_b = 1.0
S_b = 1.4

FIG. 3



$\frac{d}{D}$
10
8
6
4
2
0
5
10
15
20
25
30
35
40
45
$\phi = 25°$
$\phi = 35°$
MAXIMUM STABLE SLOPE ANGLE -α-

FIG 4

The slope in the example for factor of safety of 1.0 would be 1 vertical to 4.94 horizontal or 12.7 degrees. Common practice for a material with a strength of $\phi = 23^o$, C = 0. (which would be a loose, cohesionless silt more than a clay) would be to consider a slope of 11.5^o (one-half of ϕ) safe for a state of steady seepage. At the same time, the maximum slope which would be considered suitable for such material is 23^o (ϕ). The addition of a few feet of select granular material on a long slope would not improve the over-all stability of the slope against a general failure. It may be concluded that the procedure for checking the stability of the slope surface under steady seepage is suitable only for cohesionless materials and not for clays. The mass stability of a clay slope must be considered rather than the stability of single particles on the slope.

ABDEL-AZIZ I. KASHEF[4], A.M. ASCE.—Mr. Karpoff presented in his paper two main parts: the first was the determination of the phreatic line through earth dams and the second deals with the slope stability. It is the writer's purpose to discuss only the first of these two parts.

The seepage of water through dams under the steady state condition is governed by the Laplace equation 2. The mathematical solution of this equation for such problems, where gravity is the essential driving force, entails some difficulty. A very powerful solution, although rather difficult, is the method of Hodographs developed by Hamel[5]. Some mathematical approximations are in current use owing to the difficulty involved in exact mathematical solutions[6]. Among these approximations are Dupuit's assumptions which are referred to in some literature as the Dupuit-Forchheimer Theory. The Pavlovsky's Theory presented by the author is also based on these assumptions. There are some objections to applying these assumptions to dam problems as they fail to give the true shape of the phreatic line and are not valid for either the entrance or the exit conditions of this line. In water-table well problems where a phreatic line exists, it has been concluded[5] that the failure of the Dupuit-Forchheimer Theory to take into consideration the surface of the line of seepage at the well should alone invalidate its use for determining the shape of the free surface. That some engineers are in favor to applying this theory, is because of its simplicity and the surprisingly exact value of the flux that can be determined by applying it. Experiments [5,7] showed that Dupuit's assumptions give the actual distribution of the piezometric heads at the impermeable base but do not give the true free surface. Accordingly, Dupuit's equation is occasionally referred to as the "base-pressure equation". Jacob concluded[8] that Forchheimer's equation for steady-state conditions of water-table wells is valid in cases where the ratio of the depth to the lateral extent of the aquifer is small and where the range of depth of flow is also not large. He stated that

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4. Lecturer, Ibrahim Univ., Cairo (Egypt); formerly, Research Fellow, Purdue Univ., Lafayette, Ind.
5. Muskat, M., "The Flow of Homogeneons Fluids Through Porous Media," McGraw-Hill, 1937.
6. Casagrande, A., "Seepage Through Dams," Journal of New England Water Works Association, pp. 131-172, 1937.
7. Babbitt, H.E. and Caldwell, D.H., "The Free Surface Around and Interference Between, Gravity Wells", Eng. Expt. Sta., Univ. of Ill., Bull. 374, 1948.
8. Jacob, C.E., "Flow of Ground Water," Chapt. V in "Engineering Hydraulics", edited by H. Rouse, John Wiley, 1950.

these restrictions are, in general no more severe than those that apply, for example, to the approximate theory of flexure for beams and concluded: "In fact, the engineering theory of beams bears somewhat the same relation to the theory of elasticity as the Forchheimer Theory of steady unconfined flow does to the more exact theory that may lead to solutions of the Laplace Equation with explicit satisfaction of boundary conditions"[8].

The writer has solved the author's example by the numerical method—iteration procedure—which proved to be a very powerful and simple method and it does not involve any doubtful assumption. The numerical methods have been recently introduced in solving seepage problems. Shaw and Southwell[9], in 1941, solved dam problems by the relaxation method. In 1949, Yang[10] solved dam and water-table well problems by the same method. The Yang solution proved that Dupuit's assumptions do not give the actual distribution of piezametric heads at the base. The same conclusion was arrived at by the writer[11] and others in dealing with water-well problems. The relaxation method was also applied by Van Deemter[12] in solving a tile drainage problem with a phreatic line. The relaxation method, though simple, requires good judgment and experience. The iteration method, however, is more systematic and far simpler for an experienced person than the relaxation procedure. In the writer's solution, iteration has been applied in its simplest and lengthy form without introducing any expiditing techniques such as "block iteration"[13]. In spite of this, it has been found that the procedure in this way is not laborious.

A coarse network is laid on the dam cross section, Fig. 1. Any node on this network is named as the given numbers of the horizontal and vertical lines passing through that node. Thus, for example, the point of intersection of the upstream water level with the upstream slope of the dam section is denoted as node 1-18 and so on. A dashed line between two nodes means that any of these two nodes will be affected by only one half the value of the other node potential. Care should be given to the selection of dashed lines in the upper boundary. The initial values of the piezometric heads at the various nodes have been roughly assumed and the iteration process has been carried for only three traverses. All head values have been multiplied by ten so as to avoid writing down the decimals. After this first stage of the coarse network, it has been proceeded to a finer net, Figure 2, the size of which is one half the coarse net. The values of the new nodes in the finer net are determined from the values of the original coarse net nodes as usual.[12,13,14] The upper shape

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9. Shaw, F.S. and Southwell, R.V., "Relaxation Methods Applied to Engineering Problems, VII, Problems Relating to Percolation of Fluids Through Porous Materials," Proc. Roy. Soc. London, Series A, vol. 178, pp. 1-17, 1941.
10. Yang, S.T., "Seepage Toward a Well Analyzed by the Relaxation Method", Doctorate Thesis, Harvard University, 1949.
11. Kashef, A.I., Touloukian Y.S. and Fadum, R.E., "Numerical Solutions of Steady-state and Transient Flow Problems", Eng. Expt. Sta., Bull. No. 117, Purdue Univ. U.S.A., July 1952.
12. Van Deemter, J.J., "Results of Mathematical Approach to Some Flow Problems Connected with Drainage and Irrigation", Applied Scientific Research, vol. A2, No. 1, pp. 33-53, Netherlands, 1949.
13. Frocht, M.M., "Photoelasticity", vol. II., Chaps. 8 and 9, John Wiley, 1948.
14. Grinter, L.E., "Numerical Methods of analysis in Engineering," McMillan, 1949.

of the finer network is selected according to the deduced free surface of the first net. Of course, this first stage could be dispensed with if reasonable initial values[11] were assumed. The phreatic line in the finer net is determined[10,11,13] from the head values at the upper nodes taking into consideration that the position head should always equal the piezometric head at this line. This condition should also be satisfied for any point at the downstream surface in the saturated zone above the downstream water level. The iteration procedure, for any traverse, has been carried on from upper towards lower horizontal lines of the net successively; at each line, the order of successive improvements has been taken from left to right. The iteration patterns for the various nodes are given in Figures 3 and 4. The iteration method of differences[13] has been applied by iterating the differences between the head values after the first traverse and those of the initial values. In this solution, the iterative process terminates after five traverses when it has been found that the differences do not exceed $\pm$ 0.4 ft. The result of the head value at node IV-31, Figure 2, is 50.4 ft. while the position head of the point on the downstream slope directly above this node is 50.9 ft.; thus point IV-31 may be regarded as the discharge point. However, for more accurate determination of the phreatic line and the discharge point, one should advance to a more finer net which can be done only for part and not for the entire section.

Table I gives the iteration procedure and results for the upper nodes; while Table 2 gives the results of the head values at the other nodes after the end of the fifth traverse. The iteration procedure may preferably be made on the drawing sheet rather than in a tabulated form. Tables are given in this discussion only for convenience. Table 3 includes a comparison between the author's solution using Pavlovsky's Theory and the iteration solution applied in this discussion. The maximum percentage difference, with respect to the head difference between the upstream and downstream levels, is about 14.5% Although this difference is high, the differences in other examples are of unknown magnitudes because this example cannot be regarded as that leading to the maximum deviation.

It is clear that the simplicity and accuracy of the iteration procedure should invalidate the application of approximate methods as that presented by the author. Moreover, the iteration method determines not only the phreatic surface but also the piezometric heads anywhere in the saturated body of the dam. It should be noted that iteration procedures do not involve any approximations as those of Dupuit's, and the results become more accurate as the nets are selected very fine which may be unwarranted in most of the cases. Pavlovsky's Theory and others cannot be applied to anisotropic soils, in which case the iteration method proved[15] to be simple and powerful. This shows clearly that any theory that involves Dupuit's assumptions as that presented by Mr. Karpoff, can be regarded as merely an approximate method for the determination of the phreatic line and cannot be applied either for anisotropic soils or for the determination of the head distribution in the saturated zone below the phreatic line.

15. Kashef, A.I., "Seepage Through Anisotropic Soils", Journal of Civil Engineering, Cairo, April 1954.

T A B L E 1

Iteration Solution of the Dam Problem (Fig. 2)

— Upper Boundary Nodes —

Node	a	b	c	d	e	f	g	h	j
II-19	821	817	- 4	- 3	-2	-1	-1	-11	810
II-20	793	792	- 1	- 2	-1	-2	-2	- 8	785
II-21	765	765	0	0	-4	-2	-2	- 8	757
II-22	751	749	- 2	- 7	-1	-2	-2	-14	737
II-23	723	722	- 1	+ 6	-1	-2	-2	0	723
II-24	698	721	+23	- 1	-2	-3	-3	+14	712
III-25	713	711	- 2	-13	-7	-5	-4	-31	682
III-26	723	689	-34	-11	-7	-5	-4	-61	662
III-27	675	664	-11	- 9	-6	-5	-3	-34	641
III-28	645	637	- 8	- 6	-5	-4	-3	-26	619
III-29	609	606	- 3	- 5	-2	-3	-2	-15	594
IV-30	539	550	+11	- 3	-2	-1	0	+ 5	544
IV-31	515	507	- 8	- 2	-1	0	0	-11	504

a : initial assigned head values; b : head values after first traverse; c = (b - a); d, e, f and g : differences after 2nd, 3rd, 4th and 5th traverse successively; h = (c + d + e + f + g); j : final result of head value = (a - h).

TABLE 2

Final Results of the Iteration Solution of the Dam

Problem (Fig.2)

"Head Values at the Nodes other than the Upper & Lower Boundary Nodes"

Node	Head Value	Node	Head Value	Node	Head Value	Node	Head Value	Node	Head Value
II-16	831	IV-15	847	V-11	878	V-31	511	VI-20	745
II-17	860	IV-16	828	V-12	868	V-32	481	VI-21	728
II-18	847	IV-17	814	V-13	858	V-33	463	VI-22	710
III-13	892	IV-18	798	V-14	847	V-34	453	VI-23	691
III-14	882	IV-19	799	V-15	833	VI- 4	900	VI-24	677
III-15	866	IV-20	760	V-16	815	VI- 5	897	VI-25	659
III-16	839	IV-21	742	V-17	799	VI- 6	895	VI-26	643
III-17	832	IV-22	726	V-18	784	VI- 7	891	VI-27	623
III-18	818	IV-23	710	V-19	768	VI- 8	889	VI-28	599
III-19	795	IV-24	692	V-20	750	VI- 9	883	VI-29	572
III-20	772	IV-25	675	V-21	732	VI-10	876	VI-30	547
III-21	751	IV-26	657	V-22	716	VI-11	869	VI-31	517
III-22	732	IV-27	635	V-23	701	VI-12	859	VI-32	492
III-23	719	IV-28	610	V-24	684	VI-13	849	VI-33	473
III-24	703	IV-29	581	V-25	668	VI-14	838	VI-34	461
IV-10	895	IV-32	455	V-26	650	VI-15	823	VI-35	455
IV-11	891	V-7	897	V-27	628	VI-16	808	VI-36	453
IV-12	882	V-8	893	V-28	602	VI-17	791	VI-37	451
IV-13	873	V-9	886	V-29	576	VI-18	776		
IV-14	863	V-10	882	V-30	545	VI-19	761		

T A B L E 3

Comparison between Pavlovsky's Theory and Iteration Solutions for the Dam Problem (Fig. 2)

$a_o = 7.0$ ft. (Pav.) $S = 162$ ft. (Pav.)

$a_o \cong 5.4$ ft. (Iter.) $S = 165$ ft. (Iter.)

x	Piezometric Head		Difference
	Pav. (in feet)	Iter. (in feet)	(in feet)
0	72.0	78.5	6.5
15	70.4	75.5	5.1
30	68.8	73.7	4.9
45	67.1	72.3	5.2
60	65.4	71.2	5.8
75	63.5	68.2	4.7
90	61.7	66.2	4.5
105	59.9	64.1	4.2
120	57.9	61.9	4.0
135	55.8	59.4	3.6
150	53.7	54.4	0.7

P.S.: a_o , x and S have the same significance as in the original paper.

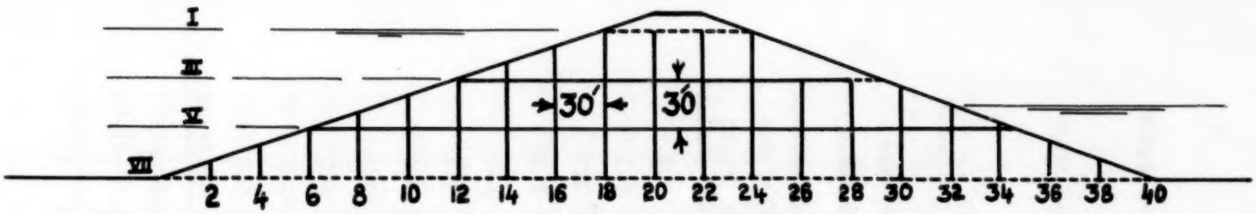

Figure 1_ Coarse Network

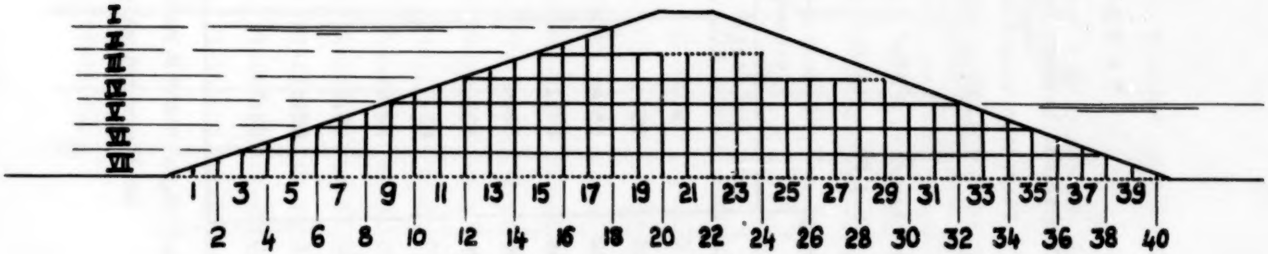

Figure 2 _Network of iterative solution

half size of the coarse network

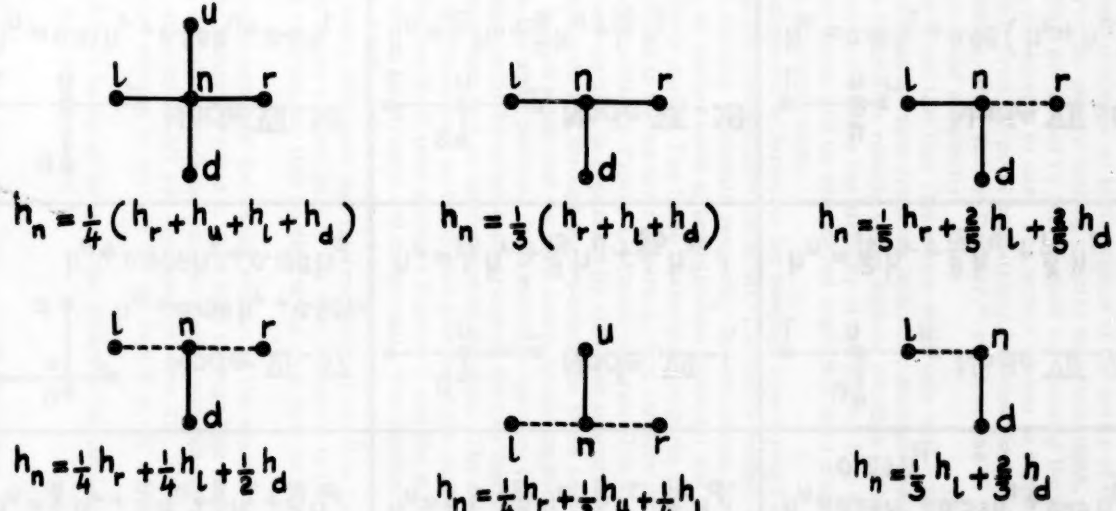

Figure 3 – Iteration Patterns: equal nodal distances

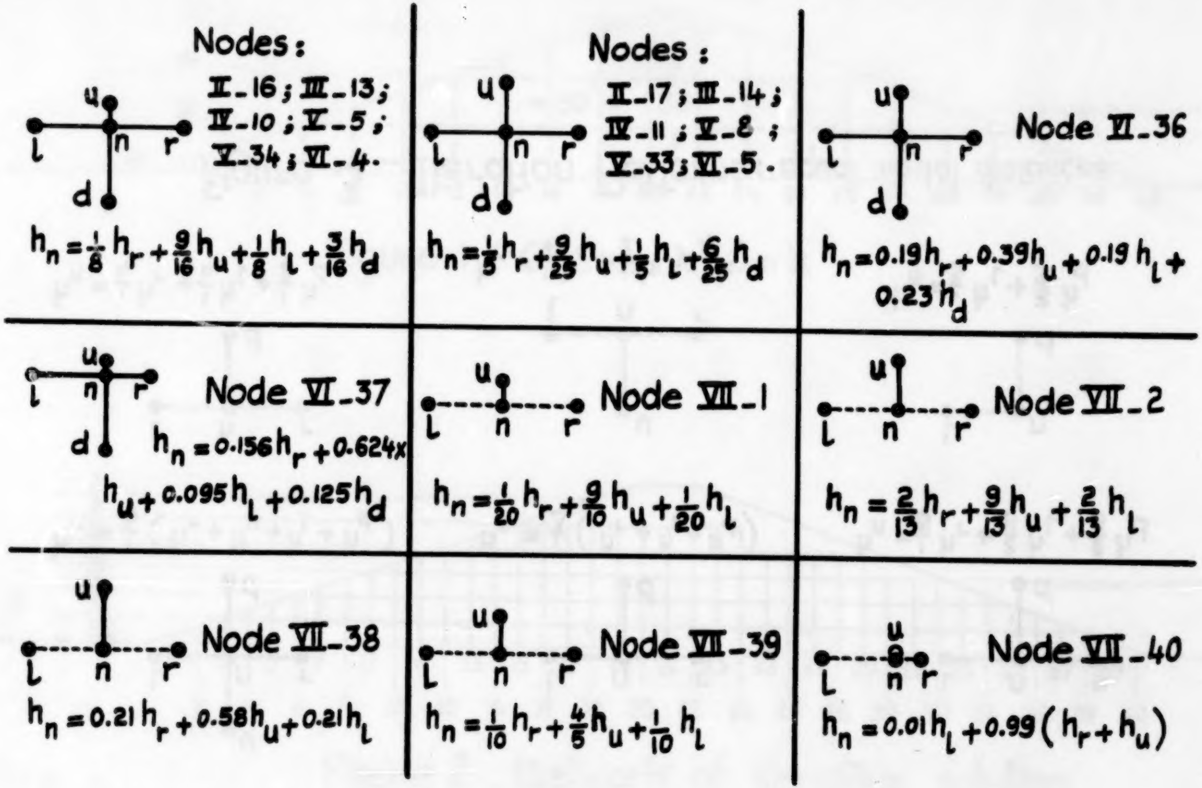

Figure 4 _ Iteration Patterns : unequal nodal distances

WILLIAM H. GOTOLSKI,[16] J.M. ASCE. - The recognized importance of determining the location of the phreatic surface in earthen hydraulic structures makes it necessary that the formulas and procedure covering the location of such surfaces be of ready application. It is realized that the exact determination of the position of the phreatic surface is not necessary; as a matter of fact, present knowledge as to the nature of the action of capillary water in this area does not yet permit making precise determinations.

The method presented by Mr. Karpoff for locating the phreatic surface is interesting and simple to use. The writer wishes to supplement Mr. Karpoff's method with a point-by-point comparison between the author's method and another procedure, after Robert Dachler,[17] which seems to have similar advantages of speed and simplicity.

It will be noted that this discussion deals only with embankments on impervious foundations because from the hydraulic standpoint an embankment on an impervious foundation results in the highest position of the phreatic line which in turn tends to decrease the stability of the slope. It is also pointed out that the tailwater depth will be considered negligible here since the tailwater depth in practice is usually zero.

The development of formulas for seepage flow through an earthen dam and for locating the phreatic surface in Mr. Dachler's method and the author's method are similar in certain respects. In both cases, for purposes of simplification, an embankment of homogeneous material and of unit width, is divided into three parts. The three characteristic parts of Mr. Karpoff's method may be found in the author's Figure 1. Mr. Dachler's cross-section is to be found in the writer's Figure 1. These separate flow systems, into which the dam cross-section is separated by different approximations, represents one hydraulic unit, and through each portion the same amount of seepage, q, occurs. It is also noted that the fluid heads should be continuous in passing from one to the other portion. Both methods agree on this requirement.

In making a point-by-point comparison of the two methods, let us proceed from the upstream part or inflow part of the dam. The author's method considers the upstream part as being that portion of the dam between the upstream slope and Section W-W (see author's Figure 1) and the streamline are assumed therein to be straight and horizontal. The Section W-W was chosen as a boundary of this portion because of geometric simplicity and laboratory experiments which indicated that the transition of the flow line from this upstream part to the central part of the dam occurred in the vicinity of this section.[18] The equation for this portion of the dam may be found in the author's paper.

Mr. Dachler assumed the inflow section of the system to be bounded by ABCD in the writer's Figure 1. The flow through this region is computed by means of a conjugate-function transformation suitable for a wedge of angle $\underline{A}$, bounded by streamline AD and equipotential line AB. BC is a streamline leaving B, with no attempt to correct for the fact it is not a free surface streamline.

If $\underline{A} = 2\pi/n$, n being an integer, the complex-variable transformation that is suitable may be expressed as

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16. Instructor of Civ. Eng., The Pennsylvania State Univ., State College, Pa.
17. "Grundwasserstroemung," by Robert Dachler, Julius Springer, Vienna, 1936.
18. U.S. Bureau of Reclamation Translation, Tech. Memo. No. 383. June 1934.

$$H + i\,\Psi = (x + iy)^n, \tag{1}$$

where the velocity potential has been replaced by the fluid head H. This substitution is for convenience only.

The location of the potential CD can be made definite by requiring the loss of energy by friction in passing through ABCD be equal to that loss in passing through ABE. This loss of energy experienced by the fluid in passing between two streamlines separated by $\Delta\Psi$ across the drop ΔH in the fluid head is

$$dW = \gamma g\, \Delta H\, \Delta\Psi, \tag{2}$$

where γ is the fluid density and g is the acceleration of gravity. The result of this condition becomes equivalent to the requirement that BCF and FDE have the same area in the (H, Ψ) plane.[19] By use of conformal mapping the resultant flux of ABCD is found to be expressed as a function of the angle $\underline{A}$ and is called the form factor:

$$f_1 = \frac{q}{k\underline{a}} = 1.12 + 1.93\tan\underline{A} \tag{3}$$

It is to be noted that $\underline{a}$ in Equation (3) refers to the potential drop only over the segment BC of the free surface.

The fundamental difference between these methods of attacking the problem of the flow in the upstream portion of the dam lies in that fact that the author's method assumes the streamlines to be horizontal and straight between the upstream slope and Section W-W, whereas Dachler uses a transformation from an area bounded by flow lines and potential lines to a triangular area for purposes of simplification. Both methods have good points in their favor for such assumptions. It is not the purpose of this discussion to argue the case for either method.

The flow to the right of BE is further resolved into separate parts, the main body of the dam and the downstream part. The usual case for the downstream part of the dam is considered here, i.e., zero tailwater.

In locating the emergence point consider the branch M_0GC (writer's Figure 2). It is assumed that the flow in this branch is horizontal. Applying Dupuit's fundamental equation, and constructing the potential line M_0C as a parabola, this permits the segment length $\underline{L}$, to be expressed as a $f(G, y_0, y)$. Consequently

$$\underline{L} = y\left(1 - \frac{y}{2y_0}\right)\tan G + y\cot G \tag{4}$$

The total discharge q may then be determined by integration and, reduced through a series formation, may be expressed as

$$q = \overrightarrow{ky_0}\,\frac{\sin 2G}{2} \qquad \text{or} \qquad f_3 = \frac{q}{ky_0} = \frac{\sin 2G}{2} \tag{5}$$

In determining the point of emergence in the author's method, it is assumed that the flow is horizontal in the downstream part (the portion between the downstream slope and Section W_0, see author's Figure 9). It may be noted that both methods assume the flow to be horizontal in this portion. The difference in the location of the point of emergence appears to vary due to the shape of the potential line which passes through the point of emergence, M_0. Pavlovsky assumes the line to be straight and Dachler uses a parabolic curve. It

19. "The Flow of Homogeneous Fluids Through Porous Media," by Morris Muskat, McGraw-Hill Book Co., In., New York, N.Y., 1937. p. 340.

appears that the assumption of a parabolic potential line is much closer to the actual flow case as the phreatic line itself is a parabola. It follows then, from the equipotential theory, that the potential lines should be perpendicular to the flow lines. Consequently a parabolic potential line would fill this requirement more closely than would a straight line.

Dachler considered the flow in the main body of the dam to be essentially linear in character, the flow being related to the drop in fluid head across it, H_2, by

$$f_2 = \frac{q}{kH_2} = \frac{H_m H_2}{L} \qquad (6)$$

where H_m is the mean height of the phreatic line along CM_0, and L is the distance EN (see writer's Figure 1). This form factor is used to relate the main body of the dam to the inflow and downstream portions of the dam.

The extent of the central part or main body of the dam differs in the author's and writer's methods by the area bounded by Section W-W (author's Figure 1) and BE (writer's Figure 1). This is due to the initial assumption of the make up of the inflow or upstream portion of the dam. In both Mr. Karpoff's method and Mr. Dachler's,[20] by applying Darcy's law for the flow of water through homogeneous material and Dupuit's relation, the general equation for the phreatic line in the central part or main body of the dam may be derived. According to Dachler the expression is

$$\frac{q}{k} = \frac{(y_2^2 - y_1^2)}{2L} \qquad (7)$$

where q = seepage flow per unit length of dam
k[21] = coefficient of permeability
L = horizontal distance from the point of emergence, M_0, to the desired point on the phreatic line
y_1 = vertical ordinate of point M_0, above the dam base
y_2 = desired vertical ordinate above dam base.

Consequently the procedure to be used is as follows: First, angle $\underline{G}$ being known, assume a discharge q. By using Equation (5), y_0 is computed. Thus the point of emergence is temporarily located.

Then using the assumed q as above and solving Equation (3), $\underline{a}$ is determined. Since $\underline{a} + H_2 + y_0 = H$, H_2 may be found. Knowing H_2, compute H_m. Solve Equation (6) for q; lack of agreement with the assumed value of q requires a repetition of this process until agreement is reached.

With q, $\underline{a}$, and y_0 known, it is now possible to determine the position of the phreatic line in the main portion of the dam. Equation (7) is used for that purpose, and in the following form:

$$y_2^2 = \frac{2qL}{k} + y_1^2 \qquad (8)$$

It is pointed out during this discussion that there are several pertinent differences in these methods of locating the phreatic surface in a dam. The

20. "Grundwasserstroemung," by Robert Dachler, Julius Springer, Vienna, 1936, p. 90.
21. It is assumed for convenience here and throughout this discussion that the effective permeability k has the value of unity.

differences in the profiles indicate that the author's method results in large deviations from other methods. For further actual comparison and for illustrative purposes the following example is used.

Illustrative Example

Given: Height of dam - 100.0'
Freeboard - 10.0'
Headwater level - 90.0'
Width of crest - 22.0'
Upstream slope - 3:1
Downstream slope - 3:1

Find:
(1) the point of emergence of the phreatic line on the downstream slope
(2) the maximum ordinate at the intersection of the phreatic line with a.

To obtain a solution to this problem reference is made to the procedure outlined in the previous paragraphs. A q is assumed; determinations of a, H_2 and y_0 are made. A check is then made on q. This method results in the required values of q, a, H_2 and y_0.

A q of 11.5 was assumed and found to check out by use of the procedure outlined above. Thus y_0 was found to be 38.4 ft. Solving Equation (3) for a resulted in 6.5 ft. H_2 is equal to 90 minus 44.9 or 45.1 ft. Solving for H_m resulted in a check of the assumed q. Consequently the point of emergence occurs on the downstream slope at a point which is 38.4 ft above the dam base. Starting at this point and using Equation (8) the equation of the curve is

$$y_2^2 = \frac{2 \times 11.5L}{k} + (38.4)^2 \qquad (9)$$

where L is the distance from the point of emergence to the desired point on the phreatic line. The plotted data are shown in the writer's Figure 3.

Table I is a comparison showing the ordinates of the phreatic line obtained by various methods. It is noted that the author's method results in a lower position of the phreatic line than do any of the other methods plotted. It would seem then that the author's method should be questioned as to its use for making stability analyses. The writer is of the opinion that the higher the position of the phreatic line the less stable is the slope. It is thus concluded that the method giving the highest position of the phreatic line should be used when slopes are to be analyzed for stability.

To complete the phreatic line the reversed curve BC (see writer's Figure 1) is drawn perpendicular to the upstream slope at the entrance point and tangent to Dupuit's curve at the point where Dupuit's curve intersects a. This follows from the equipotential theory and the continuity of the flow.

In conclusion, the writer wishes to emphasize his belief that the assumptions of Mr. Dachler approach the actual conditions more closely than do those of the author's method. However, Mr. Karpoff has made a valuable contribution in presenting his paper, especially the mathematical approach which may be used by field engineers to make quick decisions for determining the stability of an earthen dam.

Table I—The values of x and y.

Coordinates	y in feet			
X, ft	DACHLER	L. CASAGRANDE	A. CASAGRANDE	PAVLOVSKY
85	–	–	–	27.85*
102.5	–	–	34*	33
109.5	–	37	36.5	35
115	38.4*	38	38	36
160	51	51	51	46.5
200	60	60	60	54.5
240	66.5	67.5	66.5	60.5
280	73.5	74	71.5	65.5
320	80	80	76.5	70.5
330	81.5	82.5	77.5	72.5
340	84	83.5	80.5	76.5
350	88	88	88	88

Note: The origin is taken as the toe of the downstream slope.
*y_0 or height of point of emergence above the dam base.

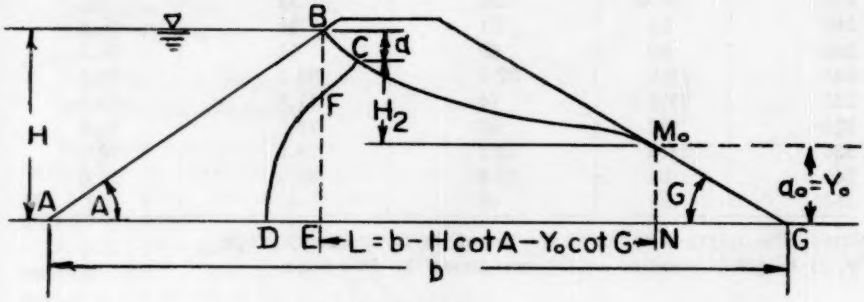

FIGURE 1

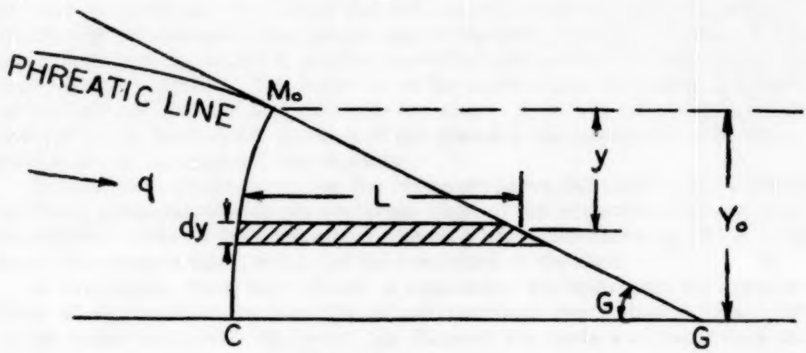

FIGURE 2

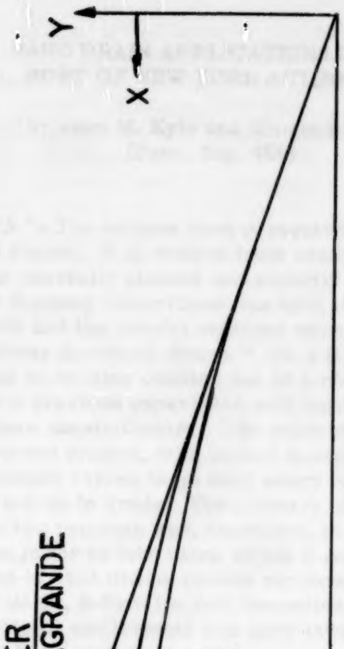

FIGURE 3

SAND DRAIN APPLICATIONS BY THE PORT OF NEW YORK AUTHORITY

by John M. Kyle and Martin S. Kapp.
(Proc. Sep. 456)

K.B. HIRASHIMA.*—The authors have presented valuable data on field experience with sand drains. It is evident from reading the paper that all of the work described was carefully planned and expertly executed.

The Territorial Highway Department has used the sand drain technique on two highway projects and the results obtained were presented before the last meeting on the Highway Research Board.[22] On a third project, sand drains were considered but were later omitted due to a change in alignment.

The Department's previous experience with highways constructed over swamp areas had been unsatisfactory. The settlements continued for years, since about 1937 for one project, with no end in sight. This has made it necessary for the maintenance forces to go back every few years, re-level and bring the road surface up to grade. The primary objective in using sand drains on the above two projects was, therefore, to accelerate the ultimate settlement, causing the latter to take place within a reasonable time interval. In this respect, the use of sand drains proved successful. Both projects were completed rather quickly, before the full theoretical settlement had taken place. But the additional settlements that have taken place since completion of the projects have been relatively small.

On both of the Department's sand-drain projects, the actual settlements with time were carefully recorded and the writer has found it interesting to compare the results with those of the authors.

It is the Department's experience that there is a distinct increase in rate of settlement immediately upon installation of sand drains. But there is a still greater rate of increase as the embankment and surcharge loads are applied.

As the embankment load increases, the pore-water pressure also increases. Usually the permeability of the soil is low so that the pore-water ordinarily cannot drain readily. Under such conditions, if the loading is increased too rapidly, a dangerously high internal hydraulic pressure will be built up, which can result in mud flows. The function of sand drains is to provide additional drainage channels so as to more quickly dissipate the high internal pore-water pressures. Even then, on contract jobs, the tendency is to apply the load too rapidly rather than too slowly. As a consequence, part of the settlement during the loading period in such cases is due, not to drainage of the pore-water, but to plastic displacement. However, after a short time interval, the plastic displacement will cease, if the overloading was moderate and practically all the subsequent settlement will be due to drainage. If the rate of loading is

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* Materials Eng., Territorial Highway Dep., Honolulu, Hawaii

22. "Hawaii's Experience with Vertical Sand Drains" by K.B. Hirashima, a paper presented at the 33rd Annual Meeting of the Highway Research Board.

very rapid, complete failure by lateral displacement of the undersoil, including the sand drains, is possible. This is a fault not of sand drains as such but of a faulty technique.

In all of the settlement curves presented by the authors, the actual settlements appear to be in good agreement with those calculated. Exact agreement is hardly to be expected in the case of soils, which as is well-known, can vary widely in physical properties from point to point.

Figure A shows a comparison of the calculated with the actual observed field settlements taken from one of the Department's projects. Prior to the installation of sand drains, a blanket of fill material was placed over the swamp to serve as a working table for the construction equipment. The time when construction work was started on the working table is taken as zero on the time axis. The observed settlements necessarily reflect the fact that the full load was not applied suddenly but over a period of time. Accordingly, for the calculated curve, the load was assumed to be applied in increments and not instantaneously. As far as long-term effects are concerned, say settlements at time exceeding six months or so after application of full load, the results are practically the same whether the loading is assumed to be instantaneous or gradual. However, for near-term effects, that is the settlements during the first few weeks after installation of sand drains, the difference is discernable. If the calculated curve had been computed on the basis of instantaneous loading, the apparent agreement between the calculated and observed curves would have been closer in this case, although Figure A is on too small a scale to show this. In connection with the Newark Airport project, the authors mentioned slides caused by too rapid loading. This was also true to some extent in the case illustrated by Figure A. From the curves, the plastic displacement caused by this rapid loading appears to have been about 0.5 ft. That is, if this plastic displacement had not occurred, the agreement between observed and calculated settlements would have been closer.

According to the data presented by the authors, the ratio of the horizontal to vertical permeability was low for all the projects covered, not exceeding two. Presumably, the values were determined by laboratory tests. The Department's experience is that such values are more easily and reliably determined by means of rather simple field tests. For the vertical permeability, the "tube" method[23] was used. A good approximation of the horizontal permeability is given by the so-called "auger" hole method.[24,25] Both methods are described in "Soil Engineering" by Spangler. However, for the auger hole method, the constants given in Spangler's book appear to the writer to give values that are too high.

The value of the paper would have been enhanced if cost data on sand drains had been included. On the Department's projects, the sand drains were 18" in diameter and the costs ranged from $1.05 to slightly over $1.26 per linear foot.[22]

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23. "A Field Method for Measuring the Permeability of Soil Below A Water Table" by Richard K. Frevert and Don Kirkham; Proc. Highway Research Board, 1948, Vol. 28, pp. 433-442.
24. "Theory of Seepage Into Auger Holes" by Don Kirkham and C.M.H. Van Bavel; Proc. Soil Science Society of America, 1948, pp. 75-82.
25. "Field Measurement of Soil Permeability Using Auger Holes" by C.M.H. Van Bavel and Don Kirkham; Proc. Soil Science Sciety of America, pp. 90-96.

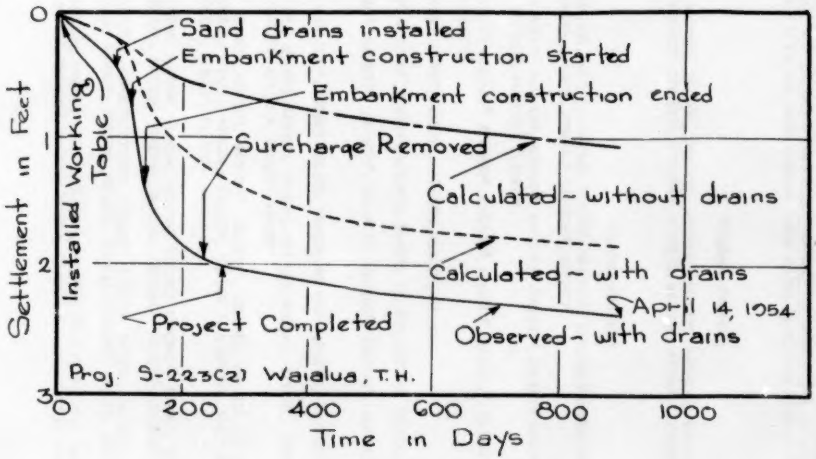
Settlement in Feet
Time in Days
0
1
2
3
0
200
400
600
800
1000
Sand drains installed
Embankment construction started
Embankment construction ended
Installed Working Table
Surcharge Removed
Calculated - without drains
Calculated - with drains
April 14, 1954
Project Completed
Observed - with drains
Proj. S-223(2) Waialua, T.H.

Figure A

PROCEEDINGS-SEPARATES

The technical papers published in the past year are presented below. Technical-division sponsorship is indicated by an abbreviation at the end of each Separate Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. For titles and order coupons, refer to the appropriate issue of "Civil Engineering" or write for a cumulative price list.

VOLUME 79 (1953)

DECEMBER: 359(AT), 360(SM), 361(HY), 362(HY), 363(SM), 364(HY), 365(HY), 366(HY), 367(SU)[c], 368(WW)[c], 369(IR), 370(AT)[c], 371(SM)[c], 372(CO)[c], 373(ST)[c], 374(EM)[c], 375(EM), 376(EM), 377(SA)[c], 378(PO)[c].

VOLUME 80 (1954)

JANUARY: 379(SM)[c], 380(HY), 381(HY), 382(HY), 383(HY), 384(HY)[c], 385(SM), 386(SM), 387(EM), 388(SA), 389(SU)[c], 390(HY), 391(IR)[c], 392(SA), 393(SU), 394(AT), 395(SA)[c], 396(EM)[c], 397(ST)[c].

FEBRUARY: 398(IR)[d], 399(SA)[d], 400(CO)[d], 401(SM)[c], 402(AT)[d], 403(AT)[d], 404(IR)[d], 405(PO)[d], 406(AT)[d], 407(SU)[d], 408(SU)[d], 409(WW)[d], 410(AT)[d], 411(SA)[d], 412(PO)[d], 413(HY)[d].

MARCH: 414(WW)[d], 415(SU)[d], 416(SM)[d], 417(SM)[d], 418(AT)[d], 419(SA)[d], 420(SA)[d], 421(AT)[d], 422(SA)[d], 423(CP)[d], 424(AT)[d], 425(SM)[d], 426(IR)[d], 427(WW)[d].

APRIL: 428(HY)[c], 429(EM)[c], 430(ST), 431(HY), 432(HY), 433(HY), 434(ST).

MAY: 435(SM), 436(CP)[c], 437(HY)[c], 438(HY), 439(HY), 440(ST), 441(ST), 442(SA), 443(SA).

JUNE: 444(SM)[e], 445(SM)[e], 446(ST)[e], 447(ST)[e], 448(ST)[e], 449(ST)[e], 450(ST)[e], 451(ST)[e], 452(SA)[e], 453(SA)[e], 454(SA)[e], 455(SA)[e], 456(SM)[e].

JULY: 457(AT), 458(AT), 459(AT)[c], 460(IR), 461(IR), 462(IR), 463(IR)[c], 464(PO), 465(PO)[c].

AUGUST: 466(HY), 467(HY), 468(ST), 469(ST), 470(ST), 471(SA), 472(SA), 473(SA), 474(SA), 475(SM), 476(SM), 477(SM), 478(SM)[c], 479(HY)[c], 480(ST)[c], 481(SA)[c], 482(HY), 483(HY).

SEPTEMBER: 484(ST), 485(ST), 486(ST), 487(CP)[c], 488(ST)[c], 489(HY), 490(HY), 491(HY)[c], 492(SA), 493(SA), 494(SA), 495(SA), 496(SA), 497(SA), 498(SA), 499(HW), 500(HW), 501(HW)[c], 502(WW), 503(WW), 504(WW)[c], 505(CO), 506(CO)[c], 507(CP), 508(CP), 509(CP), 510(CP), 511(CP).

OCTOBER: 512(SM), 513(SM), 514(SM), 515(SM), 516(SM), 517(PO), 518(SM)[c], 519(IR), 520(IR), 521(IR), 522(IR)[c], 523(AT)[c], 524(SU), 525(SU)[c], 526(EM), 527(EM), 528(EM), 529(EM), 530(EM)[c], 531(EM), 532(EM)[c], 533(PO).

NOVEMBER: 534(HY), 535(HY), 536(HY), 537(HY), 538(HY)[c], 539(ST), 540(ST), 541(ST), 542(ST), 543(ST), 544(ST), 545(SA), 546(SA), 547(SA), 548(SM), 549(SM), 550(SM), 551(SM), 552(SA), 553(SM)[c], 554(SA), 555(SA), 556(SA), 557(SA).

DECEMBER: 558(ST), 559(ST), 560(ST), 561(ST), 562(ST), 563(ST)[c], 564(HY), 565(HY), 566(HY), 567(HY), 568(HY)[c], 569(SM), 570(SM), 571(SM), 572(SM)[c], 573(SM)[c], 574(SU), 575(SU), 576(SU), 577(SU), 578(HY), 579(ST), 580(SU), 581(SU), 582(Index).

c. Discussion of several papers, grouped by Divisions.
d. Presented at the Atlanta (Ga.) Convention of the Society in February, 1954.
e. Presented at the Atlantic City (N.J.) Convention in June, 1954.

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